

# Thermodynamics with Domain Wall Fermions and anomaly in hot QCD

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# Chiral transition in QCD

$$SU_L(2) \otimes SU_R(2) \otimes U_A(1) \otimes U_B(1)$$

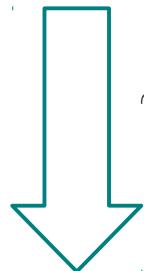
axial anomaly



$$\psi : e^{-i\alpha \gamma_5} \psi$$

$$SU_L(2) \otimes SU_R(2) \otimes U_B(1)$$

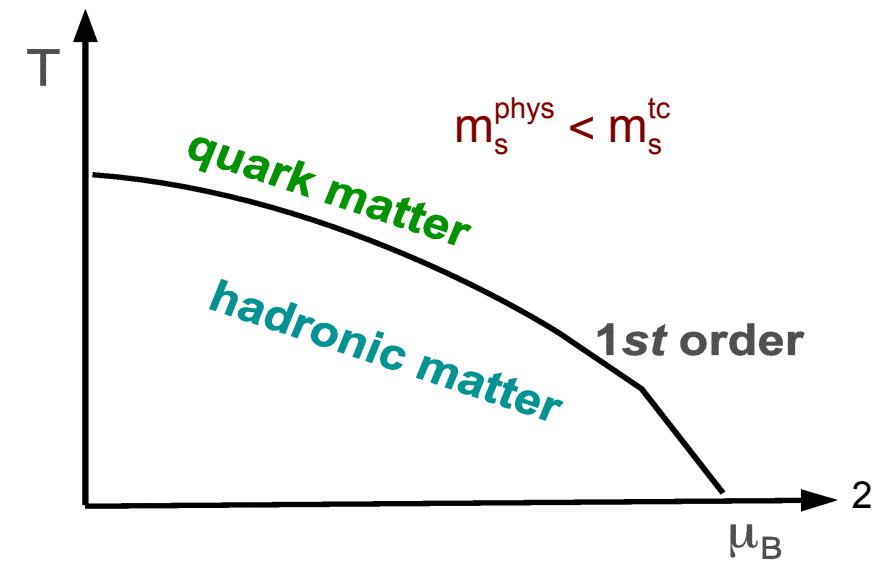
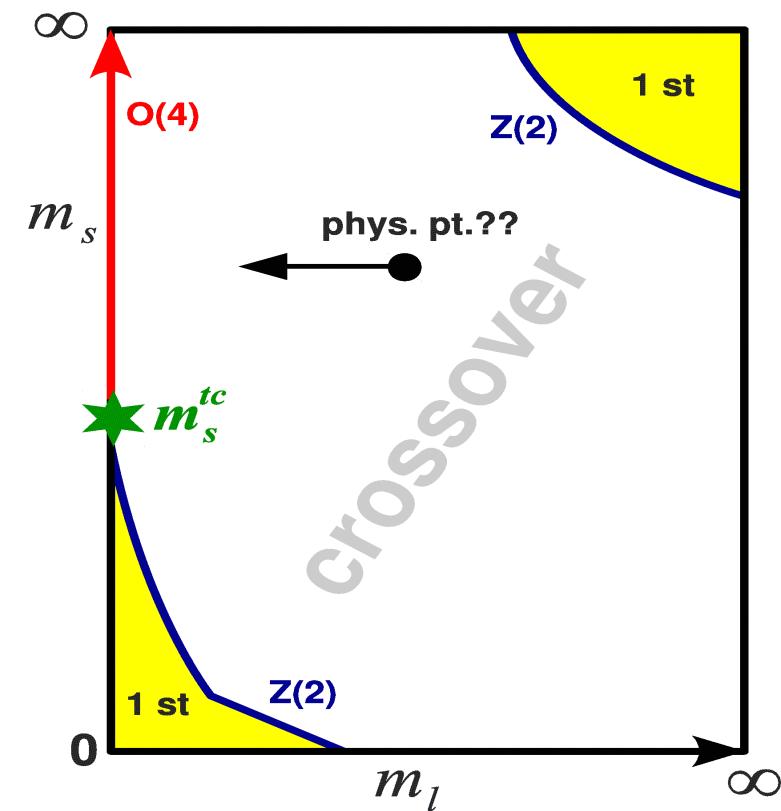
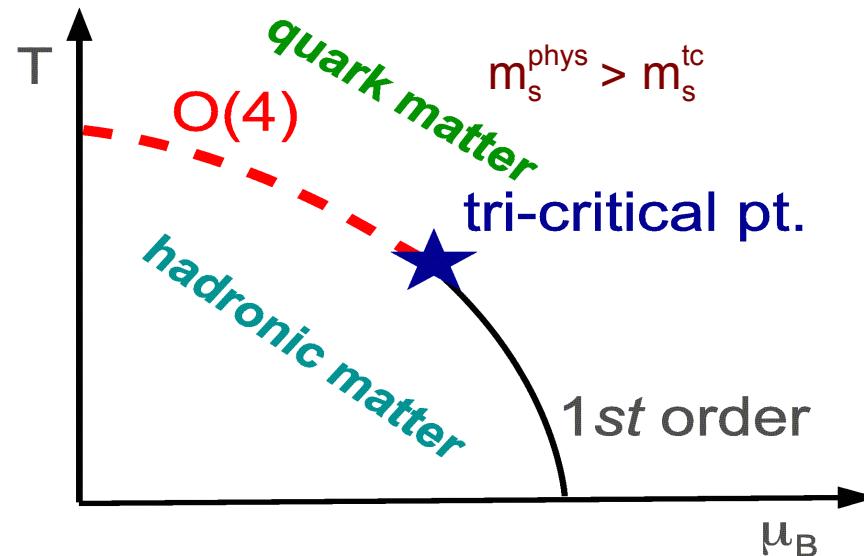
$\chi$  SM  
breaking



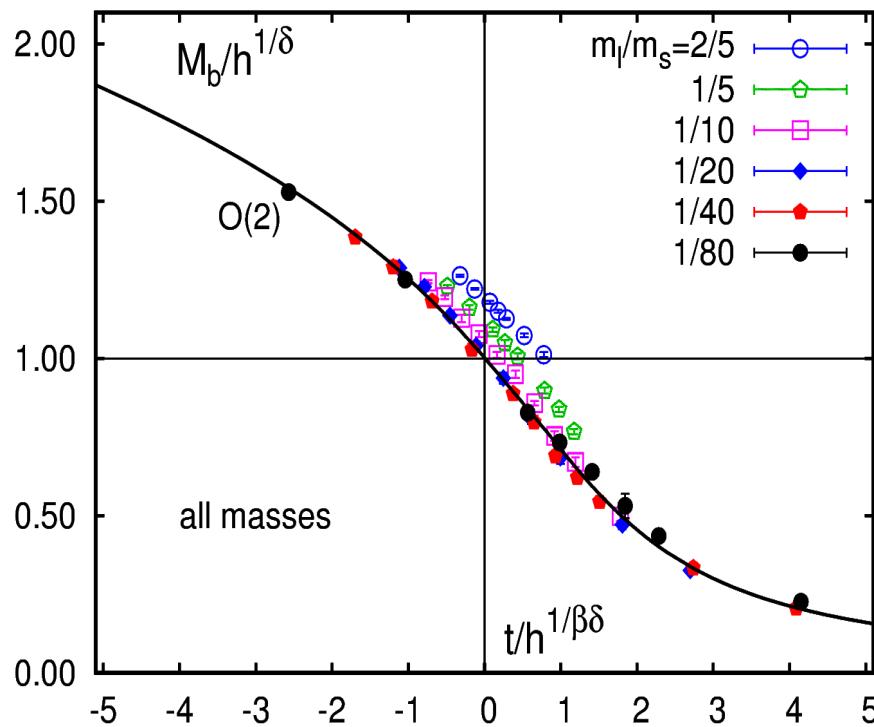
$$\psi : e^{-i\alpha \gamma_5 \tau_i} \psi$$

restores  
at high T

$$SU_V(2) \otimes U_B(1)$$

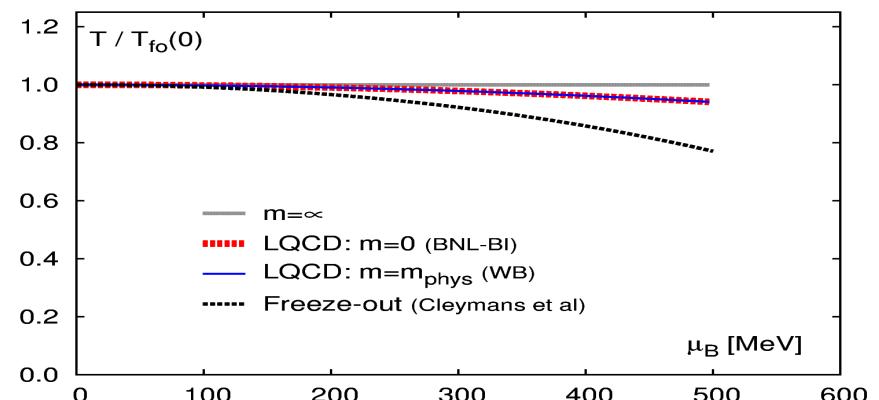
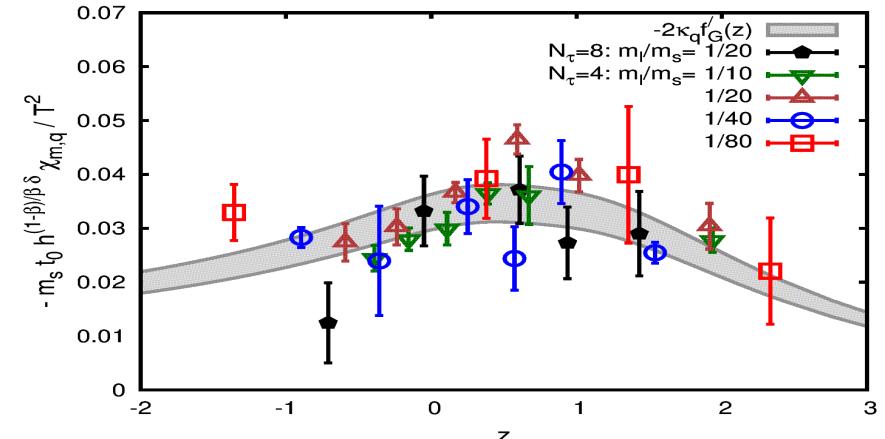


# Chiral transition in QCD



BNL-BI: Phys Rev D80, 094505 (2009)

BNL-BI, Phys Rev D83, 014504 (2011)



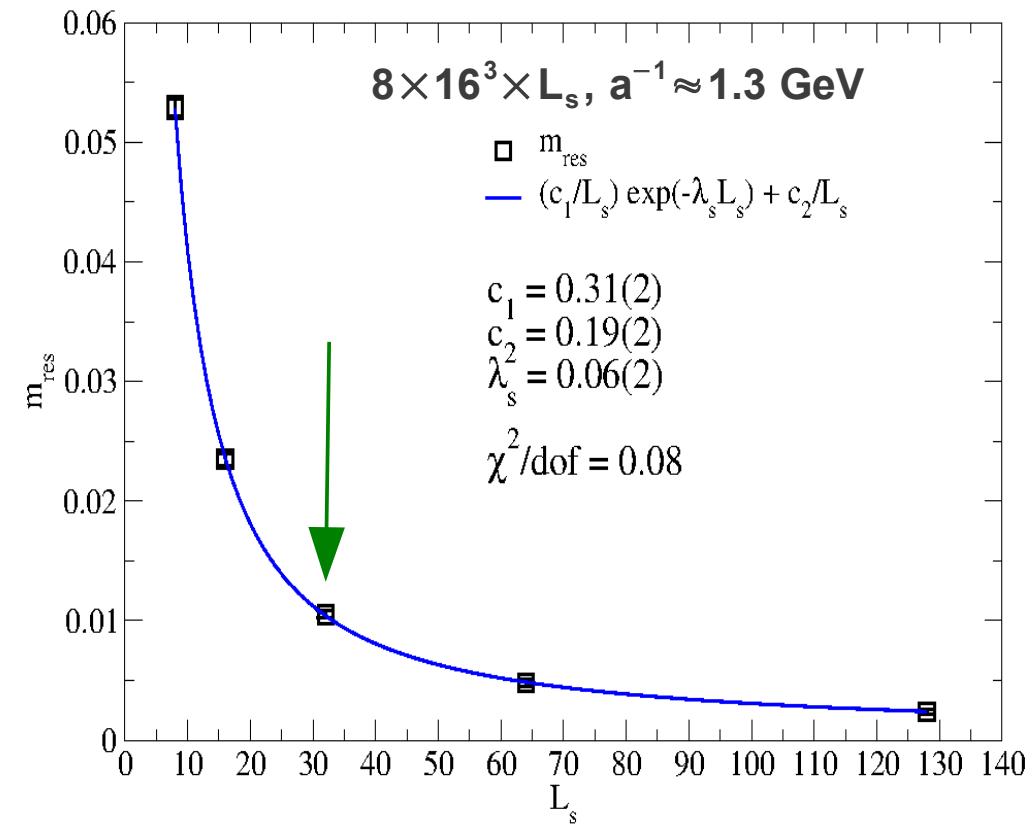
staggered fermions: intertwined chiral & continuum limit

$a \neq 0, m \rightarrow 0$ :  $O(2)$ , instead of continuum  $O(4)$

need exact chiral symmetry for  $a \neq 0$

chiral fermions: Domain Wall Fermions

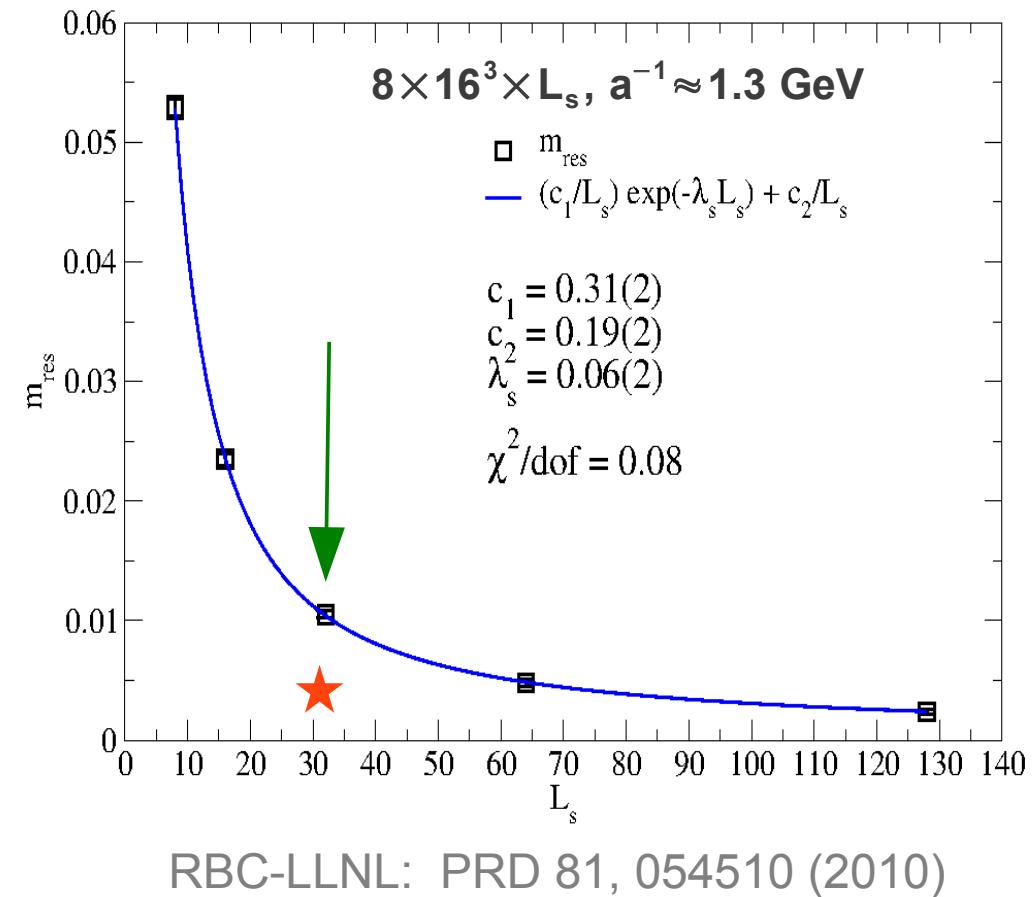
# Thermodynamics with DWF: problem and progress



✓ coarse lattice spacings:  $m_{\text{res}} \sim 1/L_s$   
instead of:  $m_{\text{res}} \sim e^{-\#L_s}$   
 $m_{\text{res}}$ : residual quark mass  
 $L_s$ : extent of 5th direction  
**large residual mass**

RBC-LLNL: PRD 81, 054510 (2010)

# Thermodynamics with DWF: problem and progress



small residual mass for coarser lattices  
and moderate extent of the 5th direction

✓ coarse lattice spacings: m<sub>res</sub> ∼ 1/L<sub>s</sub>  
instead of: m<sub>res</sub> ∼ e<sup>-#L<sub>s</sub></sup>  
m<sub>res</sub>: residual quark mass  
L<sub>s</sub>: extent of 5th direction  
large residual mass

★ DSDR

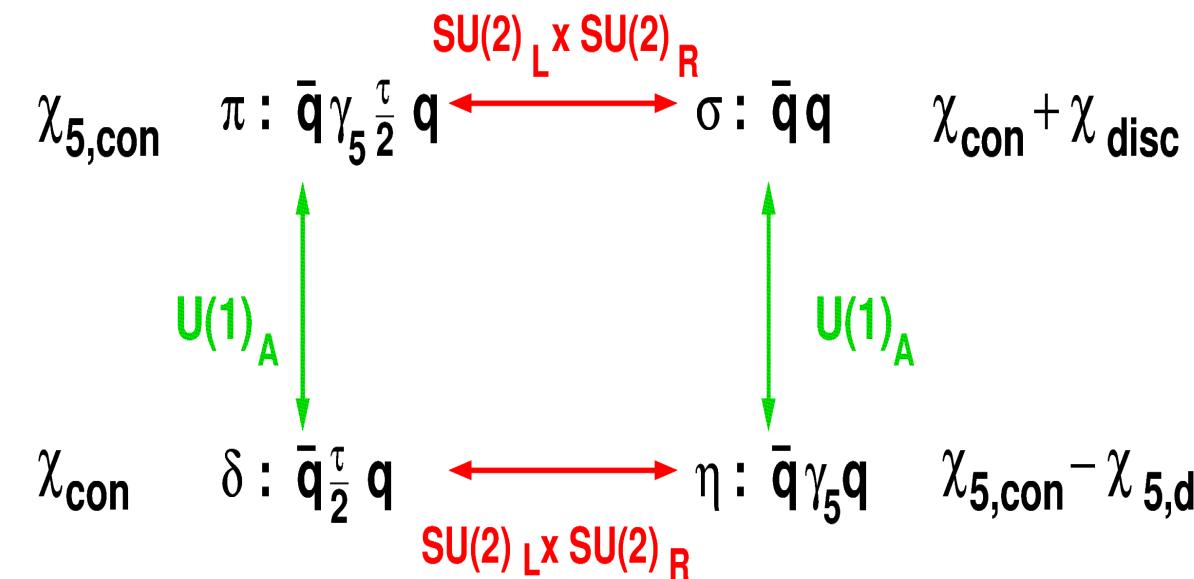
Dislocation Suppressing Determinant Ratio

$$\frac{\det [ D_w^+ (-M_5 + i\epsilon_f \gamma_5) D_w (-M_5 + i\epsilon_f \gamma_5) ]}{\det [ D_w^+ (-M_5 + i\epsilon_b \gamma_5) D_w (-M_5 + i\epsilon_b \gamma_5) ]}$$

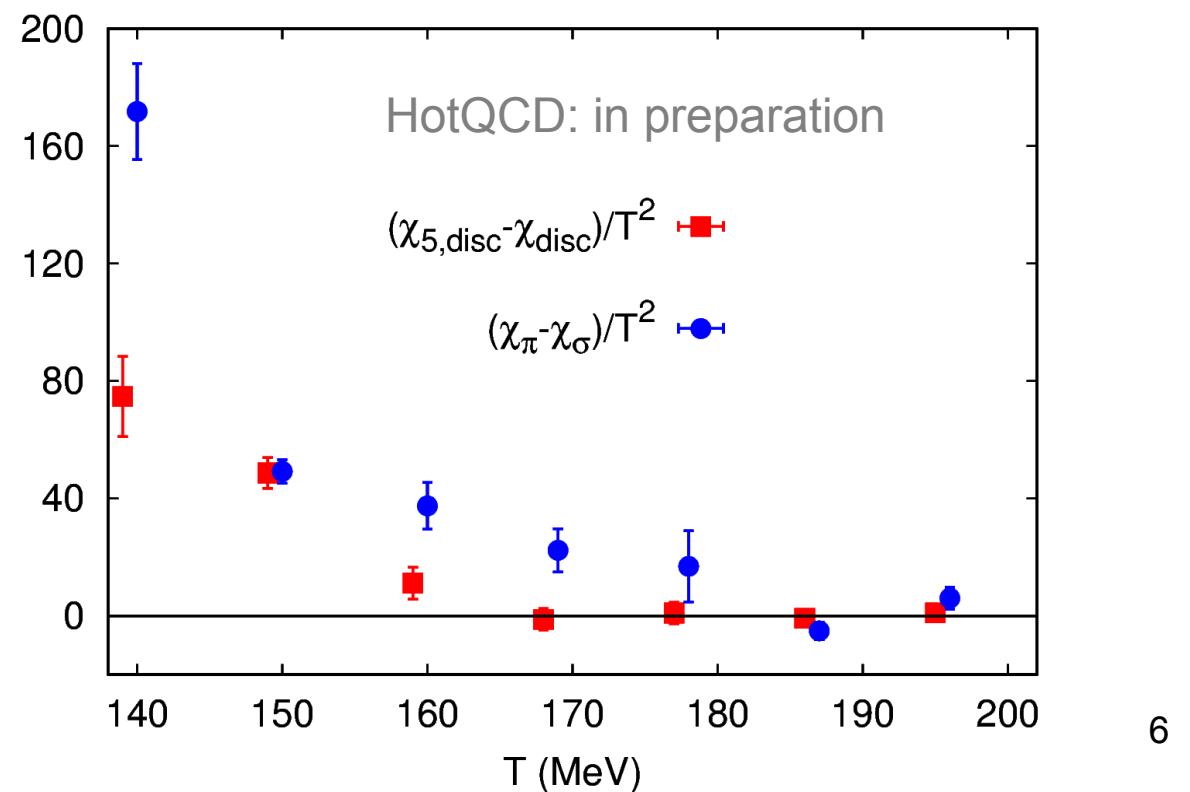
suppresses gauge field dislocations,  
but ensures topological tunneling,  
minimally affects the UV modes

- ① HotQCD: 8 $\times$ 16 $^3$  $\times$ 32(48), m<sub>π</sub>=200 MeV, m<sub>K</sub>=phys., T=135–200 MeV
- ② RBC-LLNL: 8 $\times$ 32 $^3$  $\times$ 32(48)

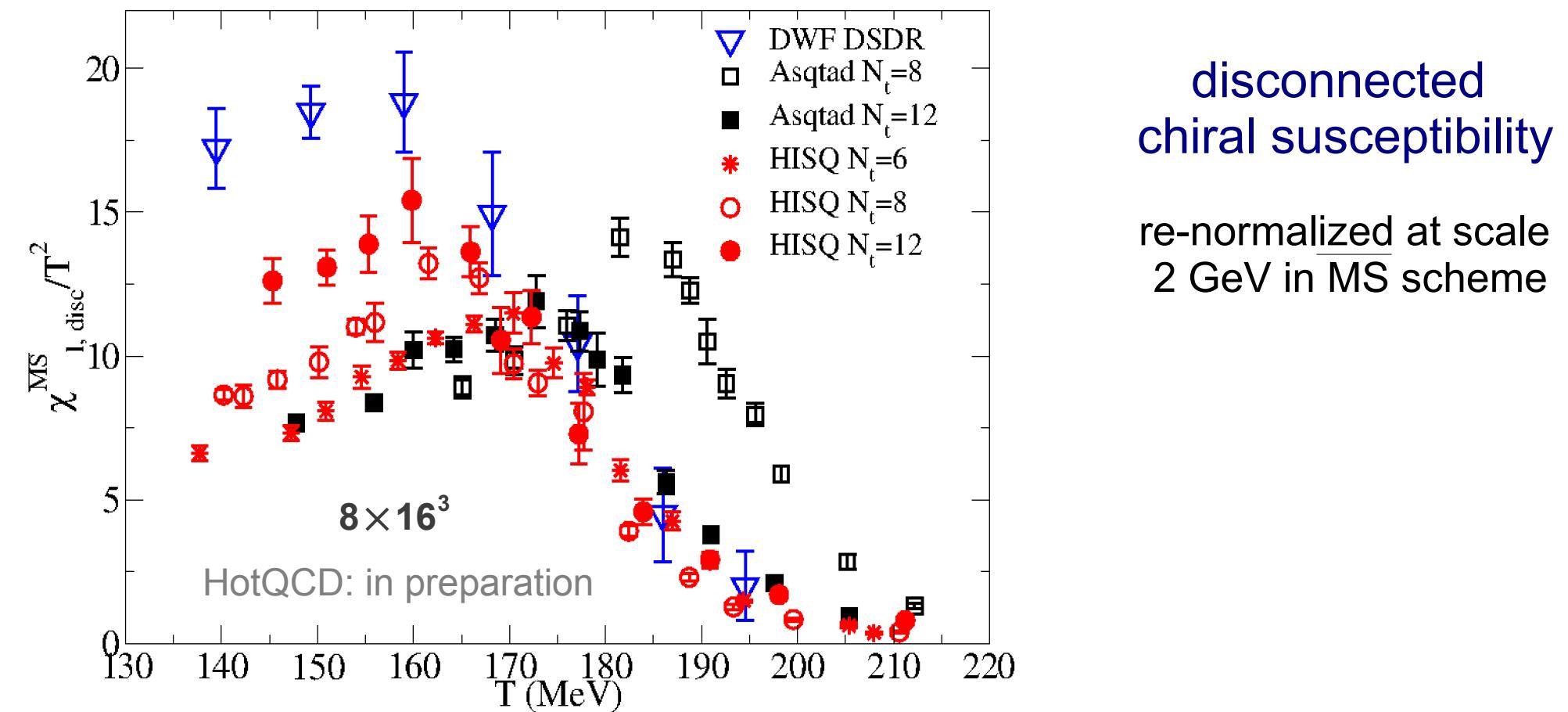
# Chiral crossover with DWF



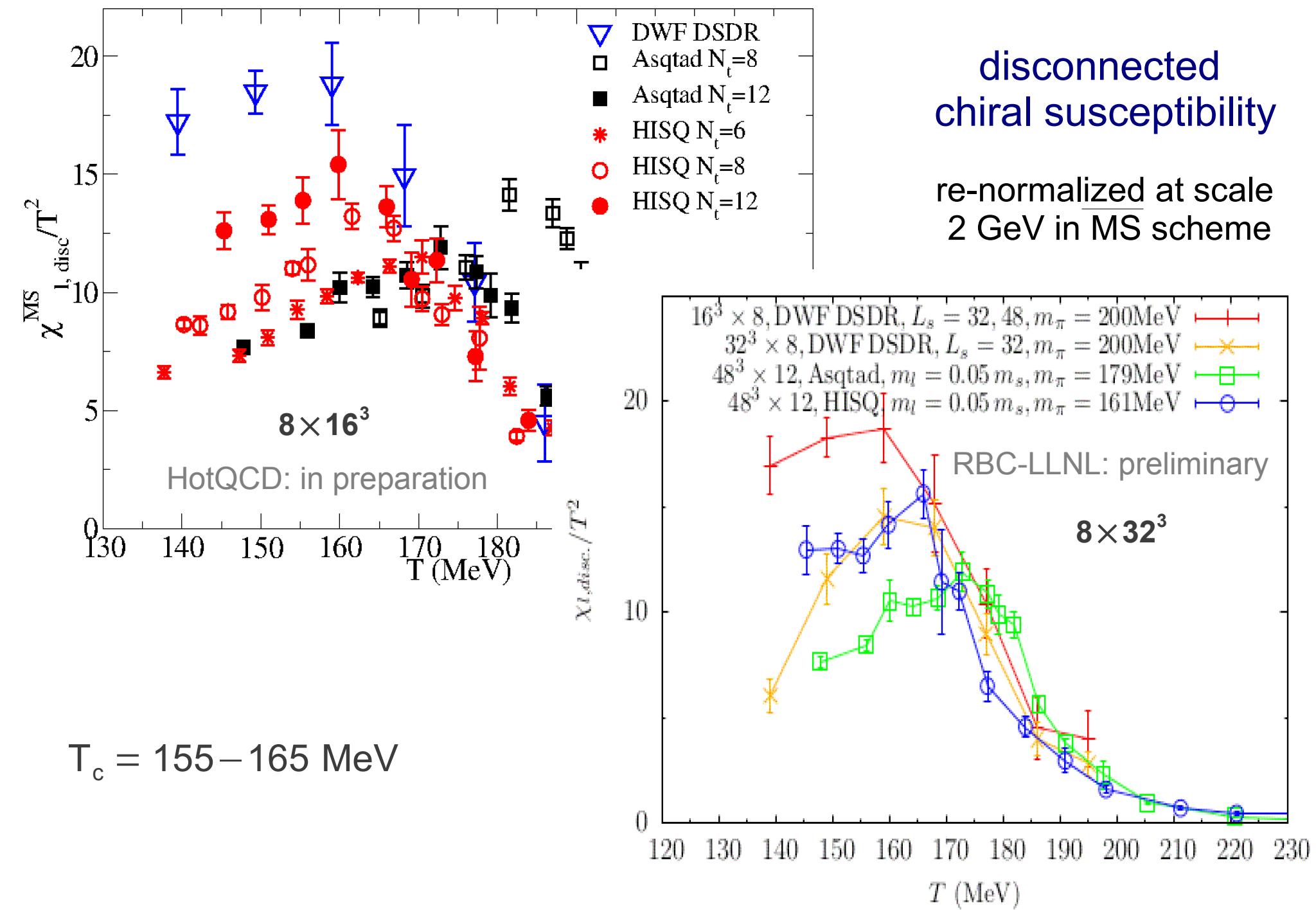
susceptibilities:  $\chi_i = \sum_x C_i(\mathbf{x})$



# Chiral crossover with DWF



# Chiral crossover with DWF

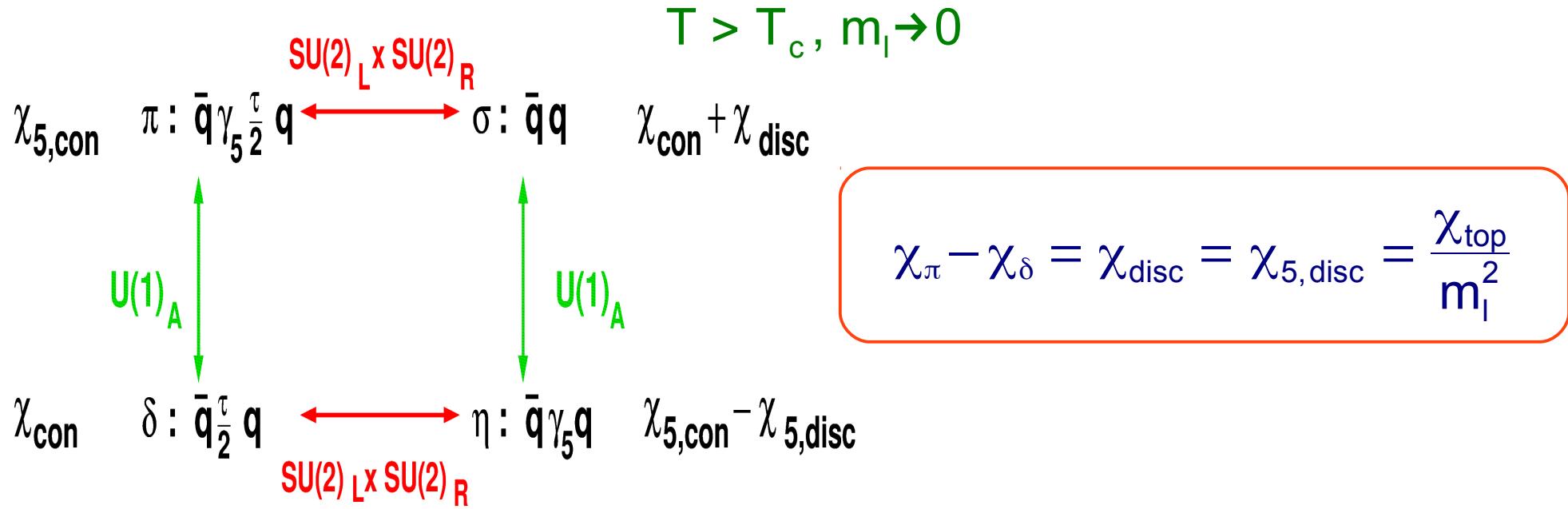


# Axial anomaly in hot QCD

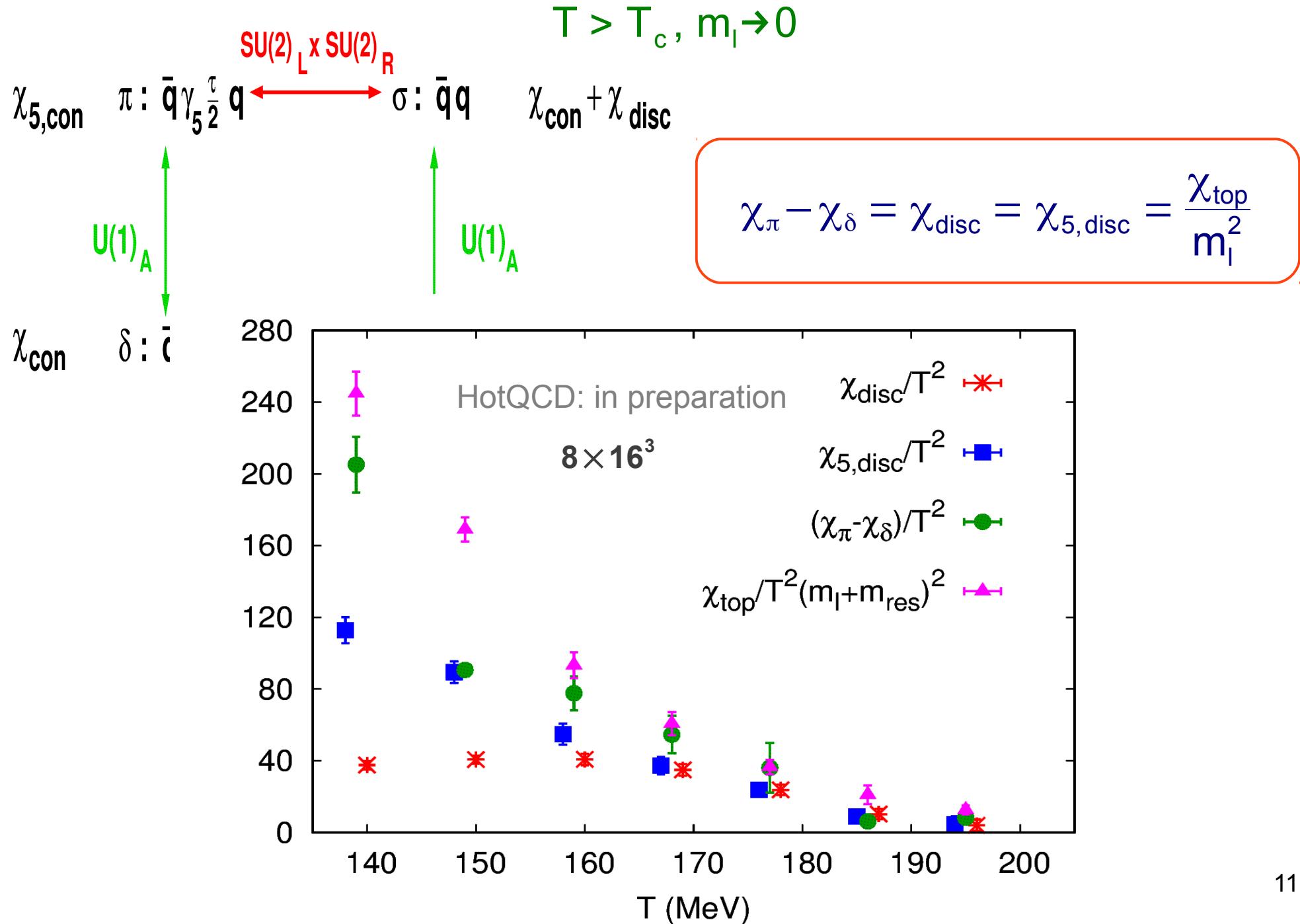
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- ✓ vacuum: non-zero  $\langle \bar{\psi} \psi \rangle$  breaks  $U_A(1)$
- ✓  $T > T_c^\chi$ :  $U_A(1)$  is broken due to presence of topologically non-trivial configurations (e.g. instantons)  
**complex non-perturbative mechanism for axial symmetry breaking ?**
- ✓ high  $T$ : exponential suppression of instanton density due to screening chromo-electric fields leads to exponentially small  $U_A(1)$  breaking, dilute instanton gas approximation

# Axial anomaly in hot QCD



# Axial anomaly in hot QCD



# Axial anomaly in hot QCD & the Dirac spectrum

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$$\chi_{\text{SM}}: \langle \bar{\psi} \psi \rangle = \int_0^\infty d\lambda \frac{2m_l \rho(\lambda)}{m_l^2 + \lambda^2} \quad U_A(1): \chi_\pi - \chi_\delta = \int_0^\infty d\lambda \frac{4m_l^2 \rho(\lambda)}{(m_l^2 + \lambda^2)^2}$$

Banks-Casher:  $\lim_{m_l \rightarrow 0} \langle \bar{\psi} \psi \rangle = \pi \rho(0)$

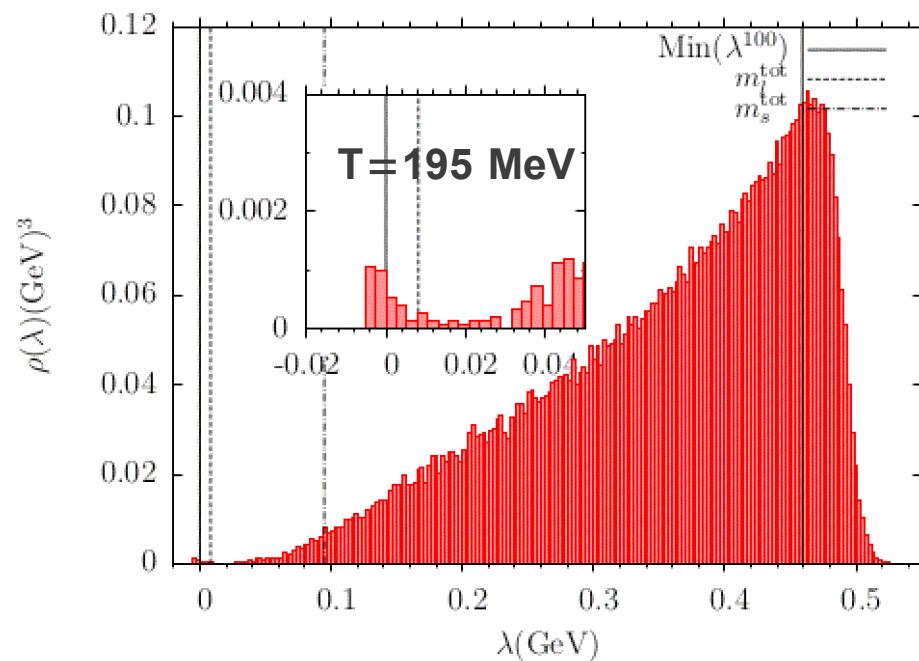
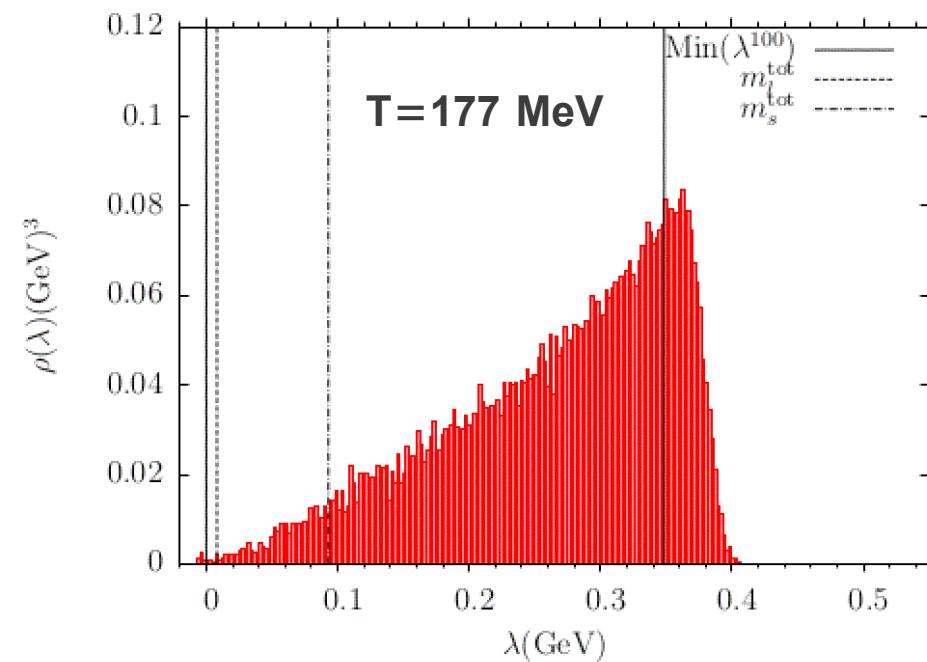
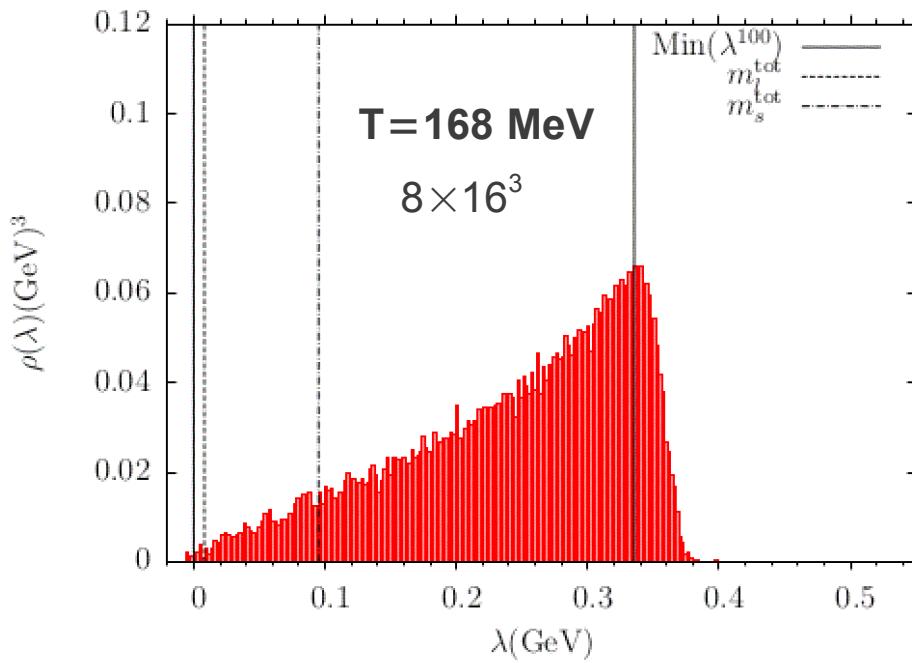
a gap in the Dirac spectrum for  $T > T_c$

$$\chi_{\text{SM}}: \langle \bar{\psi} \psi \rangle = 0 \quad U_A(1): \chi_\pi - \chi_\delta = 0 ?!$$

one obvious resolution: dilute instanton gas approximation

$$\rho(\lambda) \sim m_l^2 \delta(\lambda)$$

# Axial anomaly in hot QCD & the Dirac spectrum



no clear evidence of a gap  
or  $m_l^2 \delta(\lambda)$  behavior in  $\rho(\lambda)$

visible finite-volume and cut-off effects

HotQCD: in preparation

# Axial anomaly in hot QCD & the Dirac spectrum

(simple) mechanisms for axial anomaly  
in the chirally symmetric phase ??

$m_l \rightarrow 0$ :

$$\langle \bar{\psi} \psi \rangle = \int_0^\infty d\lambda \frac{2m_l \rho(\lambda)}{m_l^2 + \lambda^2} = 0$$

$$\chi_\pi - \chi_\delta = \int_0^\infty d\lambda \frac{4m_l^2 \rho(\lambda)}{(m_l^2 + \lambda^2)^2} \neq 0$$

$$\chi_\pi = \langle \bar{\psi} \psi \rangle / m_l = \text{finite}$$

$$\chi_{\text{disc}} = \int_0^\infty d\lambda \frac{2m_l (d\rho(\lambda)/dm_l)}{m_l^2 + \lambda^2} = \chi_\pi - \chi_\delta$$

$$\chi_\delta = \int_0^\infty d\lambda 2\rho(\lambda) \frac{d}{dm_l} \left[ \frac{2m_l}{m_l^2 + \lambda^2} \right] = \text{finite}$$

$\lambda \rightarrow 0$ :

✓  $\rho(\lambda) \sim m_l^2 \delta(\lambda)$

✗  $\rho(\lambda) \sim \lambda$

✓  $\rho(\lambda) \sim m_l$

need to confirm using  
chiral / domain wall fermions

requires many quark masses  
and larger volumes

# Summary

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- ✓ chiral fermions are essential for deeper insight into the nature of the QCD chiral transition
- ✓ chiral fermions are required to understand the mechanism of the axial anomaly in QCD
- ✓ RBC-LLNL + HotQCD have started serious efforts to address these questions using Domain Wall Fermions
- ✓ immense technical progress for DWF thermo
- ✓ more physics results coming soon ...